2. *Llama.* Pectineus Muscle.

		in.
a.	Length of pubic origin	1·91
b.	„ femoral insertion	3·50
c.	„ anterior fibre	3·05
d.	„ posterior fibre	6·91
e.	First diagonal	4·09
f.	Second diagonal	6·77

Hence we find

$$\begin{array}{rl} ab = 1\cdot91 \times 3\cdot50 = & 6\cdot68 \\ cd = 3\cdot05 \times 6\cdot91 = & \underline{21\cdot07} \\ ab + cd \; \ldots\ldots = & 27\cdot75 \\ ef = 4\cdot09 \times 6\cdot77 = & \underline{27\cdot69} \\ \text{Difference} = & 0\cdot06 \end{array}$$

This difference amounts to $\frac{1}{462\cdot5}$ part of the whole.

III. "On some Elementary Principles in Animal Mechanics.—No. VI. Theory of Skew Muscles, and investigation of the conditions necessary for Maximum Work." By the Rev. SAMUEL HAUGHTON, F.R.S., M.D. Dubl., D.C.L. Oxon., Fellow of Trinity College, Dublin. Received April 3, 1872.

Let us suppose two bones, A B and A′ B′, not lying in the same plane, connected by muscular fibres; and through these bones let us draw any two planes intersecting in a line P Q; if the bone A B be fixed and the bone A′ B′ be movable and compelled to turn round the line P Q regarded as an axis of rotation, it is required to find the conditions necessary in order that the work done by the contraction of the muscle shall be a maximum.

A muscle such as is here imagined will form a skew surface, and no two of its fibres will intersect in space. I have succeeded in demonstrating the following propositions in the case of maximum work:—

1. The axis of rotation P Q must be formed by the intersection of rectangular planes passing through A B and A′ B′.

2. A certain hyperboloid of one sheet may be drawn passing through the bones A B and A′ B′; and the axis of maximum work is a generator of this hyperboloid belonging to the group different from that to which A B and A′ B′ belong.

3. The generator which is the axis of rotation of maximum work is found by the solution of a biquadratic equation.

4. In the muscles which are found in nature, the root of the biquadratic which fixes the axis of maximum work is always nearly equal to zero.

5. If there be *n* fibres in the skew muscle, we can draw a certain line O O′, joining two points on A B and A′ B′ (or A B and A′ B′ produced), such that a single fibre acting in the line O O′, with *n* times the force of a

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fibre of the original muscle, will produce the same mechanical effect as the whole skew muscle.

6. When the equivalent fibre O O′ is drawn, it is always found to be so placed in relation to the axis of rotation as to produce the maximum work possible.

7. Two generators can be assigned on the locus hyperboloid, dividing the surface into two regions; in one of these regions each generator is an axis of stable equilibrium, and in the other region each generator is an axis of unstable equilibrium; the generators separating the two regions are axes of neutral equilibrium. The axis of maximum work is a *unique* axis lying in the middle of the region of unstable equilibrium.

8. Every skew muscle is a *supplemental contrivance* for the purpose of procuring a line of force joining points O and O′ already occupied by other structures, so that it would be impossible to apply the force directly.

Let I I′ be the shortest line joining A B and A′ B′, and take this line for axis of z, and the lines bisecting the angles between A B and A′ B′ for axes of x and y, the origin being placed at the point of bisection of I I′.

The equation of the locus hyperboloid is

$$(y^2-m^2x^2)+(1-m^2)(z^2-c^2)=0, \quad . \; . \; . \; . \; . \; . \quad (1)$$

where m is the tangent of half the angle between the bones, c is half the line I I′.

The equations of the bones are

$$\mathrm{AB}\begin{cases} y-mx=0, \\ z-c\;\;=0, \end{cases} \quad \mathrm{A'B'}\begin{cases} y+mx=0, \\ z+c\;\;=0. \end{cases}$$

The equations of the rectangular planes passing through the bones are

$$\left.\begin{aligned} y-mx+\lambda\,(z-c)=0, \\ y+mx+\lambda'(z+c)=0, \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

where λ and λ' are parameters connected by the relation

$$\lambda\lambda'=m^2-1.$$

If x, x' be the coordinates of the extremities of any fibre whose length is p, and if we write

$$\left.\begin{aligned} &\mathrm{X}=\Sigma\left(\frac{x}{p}\right), \quad \mathrm{X}'=\Sigma\left(\frac{x'}{p}\right), \\ &\mathrm{K}=m^2\Sigma\left(\frac{xx'}{p}\right)+c^2(m^2-1)\Sigma\left(\frac{1}{p}\right), \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \quad (3)$$

then the biquadratic which determines the generator of maximum work is

$$\left.\begin{aligned} &-\mathrm{K}(m^2+1)\,.\,\lambda^4 \\ &+2c\{m\mathrm{X}(m^4+1)-m\mathrm{X}'(m^4-1)\}\,.\,\lambda^3 \\ &+2c(m^2-1)\{m\mathrm{X}(m^4-1)-m\mathrm{X}'(m^4+1)\}\,.\,\lambda \\ &+(m^2-1)(m^4-1)\mathrm{K}=0. \end{aligned}\right\} \quad . \; . \quad (4)$$

The sums (3) were calculated from measurements made on freshly dissected animals, and the biquadratic coefficients then calculated, with the following results.

1. *Llama.* Adductor secundus (β):—

$$-0{\cdot}116\lambda^4+3{\cdot}375\lambda^3+4{\cdot}903\lambda+0{\cdot}042=0\,;$$

root, $\lambda=-0{\cdot}0085$.

2. "*Master Macgrath*"*. Adductor secundus (β):—

$$-0{\cdot}479\lambda^4+2{\cdot}395\lambda^3+2{\cdot}128\lambda+0{\cdot}135=0\,;$$

root, $\lambda=-0{\cdot}063$.

3. *Woman.* Adductor secundus (β):—

$$-0{\cdot}153\lambda^4+5{\cdot}288\lambda^3+3{\cdot}710\lambda+0{\cdot}040=0\,;$$

root, $\lambda=-0{\cdot}011$.

4. *African Leopard.* Adductor secundus (β):—

$$+1{\cdot}372\lambda^4+1{\cdot}343\lambda^3+1{\cdot}091\lambda-0{\cdot}077=0\,;$$

root, $\lambda=+0{\cdot}07$.

5. *Lion.* Adductor secundus (β):—

$$-0{\cdot}647\lambda^4+5{\cdot}456\lambda^3+4{\cdot}495\lambda+0{\cdot}405=0\,;$$

root, $\lambda=-0{\cdot}09$.

The position of the points O and O′ being determined, I found, by calculations too long to give in an abstract, the relation between the work actually done by the muscle and the maximum work possible, with the following satisfactory results:—

Muscle.	Work done expressed in percentage of maximum possible.
Adductor secundus (β):—	
1. Llama	99·63 per cent.
2. "Master Macgrath" ..	98·78 „
3. Woman..............	99·86 „
4. African Leopard	97·13 „
5. African Lion	97·92 „
Adductor secundus (α):—	
6. African Leopard	99·98 „
7. African Lion..........	98·88 „
Adductor primus:—	
8. African Lion..........	97·70 „

Skew muscles are only employed in nature as a *supplemental contrivance*; and they differ from all other muscles in this respect, that whereas the *equivalent fibre* of any other muscle always coincides with some real fibre of the actual muscle, the equivalent fibre of the skew muscle does not coincide with any actual fibre, but, on the contrary, may lie completely outside the muscle. Hence the skew muscle is used to produce an additional force along O O′, where the points O and O′ are already occupied for other purposes; and in producing this force along O O′ portions of bone A B, A′ B′ at a distance, and not occupied for other purposes, are made use of, to serve as the origin and insertion of the supplemental muscle. The Table just given shows that in the employment of these supplemental muscles they are always so arranged as to work to the maximum advantage.

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* Lord Lurgan's famous greyhound.

Thus the action of skew muscles, which, on a hasty examination, would be pronounced to be *imperfections*, furnishes another proof of my postulate.

POSTULATE.—*The muscles, bones, and joints of all animals are so related geometrically to each other as to produce in every case the maximum amount of work.*

IV. "On the Rings produced by Crystals when submitted to Circularly Polarized Light." By WILLIAM SPOTTISWOODE, M.A., Treas. R.S. Received April 24, 1872.

The general optical arrangements here used are known. Particular cases of the phenomena resulting from it have been described by Fresnel and by Airy; and more have doubtless been observed by others. The main part of the apparatus consists, so far as polarization is concerned, of the ordinary polarizer P, analyzer A, and crystal plate to be examined, C. To this are added two quarter-undulation plates of mica, Q, Q_1, one of which, Q, is placed below and the other, Q_1, above the crystal C. Let i, a, b, j be the angles between the principal sections of P, Q, C, Q_1, A, taken two and two together in the order written, all the angles being considered to be of the same sign when measured in the same direction—say, positive with that of a clock-hand. Then, if θ be the retardation produced in any ray, whose wave-length is λ, by the crystal C, the intensity of the ordinary image at any point is given (Verdet, 'Leçons d'Optique Physique,' tome ii. p. 201 *) by the formula

$$\begin{aligned} I^2 = {} & \cos^2(j-i)\cos^2(a+b)+\sin^2(j+i)\sin^2(a+b) \\ & +(\cos 2i \sin 2a \sin 2b \cos 2j - \sin 2i \sin 2j)\sin^2\frac{\theta}{2} \\ & +(\cos 2i \sin 2a \sin 2j + \sin 2i \sin 2b \cos 2j)\sin\frac{\theta}{2}\cos\frac{\theta}{2}. \end{aligned}$$

Of this general case four particular instances have been studied, viz. :—

$$a = b = 45^\circ, \text{ whence } I^2 = \sin^2\left(j+i+\frac{\theta}{2}\right); \quad . \quad . \quad . \quad \text{(I.)}$$

$$a = -b = 45^\circ, \text{ whence } I^2 = \cos^2\left(j-i-\frac{\theta}{2}\right); \quad . \quad . \quad . \quad \text{(II.)}$$

$$i = j = 45^\circ, \text{ whence } I^2 = \cos^2\frac{\theta}{2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(III.)}$$

$$i = -j = 45^\circ, \text{ whence } I^2 = \sin^2\frac{\theta}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(IV.)}$$

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* From which work the greater part of the discussion of the cases I., II., III., IV. has been taken.